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Research article

MATHEMATICAL MODELLING OF POTASSIUM DEPOSITION IN NTANWAOGBA CREEK, PORT HARCOURT METROPOLIS, NIGER DELTA OF NIGERIA

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Abstract

Potassium is one of the microelement that increase the deposition of microbes in surface water, this substance also play other roles in surfaces water, point sources discharge of this pollutant were observed through thorough investigation in the creeks, but the rate of deposition need lots of scientific approach to determined there various rates of concentration at any station, these were observed critically that made it imperative to develop modeling system thus generated the governing equation. The derived solution produces other model that were also considered base on the transport system of the substances, the study expressed the final model base on several conditions that determined the deposition of potassium at any point sources in the creek, the study is imperative because it will be applied to examined the rates of concentration at any point of discharge in the study area.

Keywords: mathematical modeling, potassium, and Ntawogba creek

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1. Introduction

Karnaphuli River estuary is one of the most important estuaries in Bangladesh and hydro-biologically it is the meeting place in which fresh water from upstream is continuously mixing with salt water from Bay of Bengal. Microbiological impairment of drinking, irrigation, or recreational waters is commonly monitored using concentrations of fecal indicator bacteria (FIB). Escherichia coli are the FIB commonly used to evaluate microbiological water quality. Concentration levels of E. coli are significantly influenced by various nonpoint sources such as surface runoff, bank soils, recreational activity, and animal excreta (Geldreich, 1996; Ferguson et al., 2003; Kim et al., 2007; Servais et al., 2007; Wilkinson et al., 2006). Bottom sediments have been recognized as a major reservoir of E. coli in freshwater environments (Geldreich, 1970). Many studies indicate that sediments can harbor much higher populations of both fecal coliforms and E. coli than the overlying water column (Goyal et al., 1977; Doyle et al., 1992; Buckley et al., 1998, Crabill et al., 1999; Smith et al., 2008; Rehmann and Soupir, 2009 Md. Wahidul Alam,2013). Different scientists reported the estuarine environment is polluted owing to the continuous discharge of pollutants from the industries and sewage effluents of the cities. Pollution of water courses associated with industrial discharge and refuse from human settlements is a global problem (Joy et al., 1990). The pollutants get dispersed by turbulence, ocean currents and tidal action firstly in the estuary, then concentrated in the food chain components through microbial action or deposited in the bottom sediments (Islam, 1998). People in Chittagong city depend on Karnaphuli River for drinking or other house hold purposes. Besides large number of people including fishermen lives in both sides of the river. Many countries in the world have developed drinking water criteria and standards. Bangladesh developed the first Water Quality Standards in 1976 based on International Drinking Water Standards (WHO, 1984). According to World Health Organization (WHO, 1993) and Environment Quality standard for Bangladesh (EQSB, 1991), the standard value of total load of E. coli for drinking water is nil/ml of water. Different researchers identified different pathogenic bacteria from contaminated fish of the Karnaphuli River but the isolation of E. coli from this river has been made rarely. Considering the importance of water and sediment of the estuary in daily life, the present research work aimed to estimate the E. coli concentration in water and soil of Karnaphuli River estuary and their relation with water parameters.

2. Theoretical background

Anthropogenic impacts encompass major changes and water management troubles for certain period in the last decades. Serious water management problems are floods; these regularly expand into the Ntawogba creek although there are no sources of spring, but in creeks loss of habitat for aquatic organisms. Especially changes of the creek bed morphology, other surface such as Rivers do implement facilitated navigation; these have caused deterioration of river habitats, similar conditions are also seen in Ntawogba creeks. Main problems for drinking water quality can be attributed to nitrate and phosphate leaching from agriculture as well as to emissions of heavy metals, biocides and substances affecting the endocrine system, this contaminant discharge directly to Ntawogba creek which increases the rates water pollution of the creek. But the study carried out precisely monitor, potassium, these are base on the facts that there is predominant deposition of this substances, these were investigated in the study location, it was observed that this potassium are predominant, these has generated lots of increase in some microbial population since they fed on the substances. The creek serves as natural sewerage lines for biological and industrial wastes. In the cities there are septic tanks, open dumps and surface impoundments. The majorities of private septic tanks are characterised by open bottoms or peculiar channels, which facilitate the seepage in depth or with direct connection to the nearby creeks and streams. Large and small-scale factories are clustered within the city and have unregulated waste disposal systems. The major solid waste disposal sites of the cities are located within the city limits, thus polluting the urban environment. These waste disposal sites are open systems, with no impermeable layer and with continuous low temperature burning. As a result of the above, the surface waters and the aquifers near urban areas area highly polluted (Ganoulis, et al 2005). Base on these factors there is need for mathematical modeling approach to precisely monitor the behaviour of potassium in Ntawogba creek; the study will definitely monitor the rate of potassium migration in the study location.

3. Governing Equation

$$V\frac{\partial c}{\partial t} = Q_n\frac{\partial^2 c}{\partial x^2} + Dy\frac{\partial^2 c}{\partial y^2} - K\frac{\partial c}{\partial x} - U\frac{\partial c}{\partial y} + K_d \qquad \text{……………………} \qquad (1)$$

The expressed principal equations are developed from the generated system for potassium deposition in Ntawogba creek, Port Harcourt metropolis. The developed governing equation is expressed base of the pressured variables that allowed the deposition and migrations of potassium in the creek, various researchers has tried in various ways to monitor microelements in surface water, but there are still lots of lapses that has not been treated for prevention and management of these contaminants in our surface water, observation were from investigations carried out in the creek, the analysis show the rate of predominant deposition of potassium in the system, such condition call for thorough examination, these were carried out but to monitor their behaviour in terms of migration process in creek which has express fluctuation, this condition find it imperative for mathematical modeling approach, the system formulated developed the governing equation expressed above.

We split the above equation with respect to direction of flow and dispersion of contaminants, we have:

$$V\frac{\partial c}{\partial t} = Q_n\frac{\partial^2 c}{\partial x^2} - K\frac{\partial c}{\partial x} = -\beta^2 \qquad \text{……………………} \qquad (2)$$

$$V\frac{\partial c}{\partial t} = Dy\frac{\partial^2 c}{\partial y^2} - U\frac{\partial c}{\partial y} + K_d = -\beta^2 \qquad \text{……………………} \qquad (3)$$

$$\left.\frac{\partial c}{\partial x}\right|_{x = L} = 0 \; ; \; C(o^-) = C(o^+) = Co, \; x = o \qquad \text{……………………} \qquad (4)$$

Where *L* is the distance in *x*-direction of the creek

$$\left.\frac{\partial c}{\partial y}\right|_{y = o, \; H} = 0 \qquad \text{……………………} \qquad (5)$$

Where H is the depth of the River,

The derived expression stated the limits determined on the transport process of potassium deposition in the creek, the concept here is to determined some level of limits that are considered in monitoring the rate of concentration at various station point in the creek, the limit considering its vertical dispersion and flow rate of the pollutant are paramount, because such conditions will thoroughly express the rates of concentration at any point of discharge.

The solution for equation (1) is of the form:

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$$C(x,y,t) = C(x,t) + C(y,t) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (6)$$

We consider equation (2), using Bernoulli's method of separation of variations: -

$$V\frac{\partial c}{\partial t} = Q_n\frac{\partial^2 c}{\partial x^2} - K\frac{\partial c}{\partial x} = -\beta^2$$

$$\Rightarrow VT^1 = Q_n X^{11} - kX^1 = -\beta^2$$

$$\Rightarrow VT^1 = -\beta^2 \;\; and \;\; Q_n X^{11} - kX^1 + \beta^2 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (7)$$

$$T = A_1 \ell^{\frac{-\lambda^2}{v}t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (8)$$

$$X = A_2 \ell^{m_1 x} + A_3 \ell^{-m_2 x} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (9)$$

The derived model in [8] and [9] show the behaviour of microelements with respect to time. Velocity of creek flow were observed in this phase, the rate of velocity monitored the exponential phase of flow, it also determined the rate of concentration within some certain period of discharge from point source, most of the source of discharge are from the drains that is charnel directly to the creek. The expressed model is in exponential phase base on the rate velocity through the point of discharge in the creek.

Where $M_1 = \dfrac{U + \sqrt{U^2 - 4Dx\lambda^2}}{2Dx}$ and $M_2 = \dfrac{U - \sqrt{U^2 - 4Dx\lambda^2}}{2Dx}$

We combine equations (8) and (9), to have

$$C(x,t) = A_1 \ell^{\frac{-\lambda^2}{K}t}\left(A_2 \ell^{m_1 x} + A_3 \ell^{-m_2 x}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (10)$$

Subject (10) to equation (4), we have

$$C(x,t) = \frac{CoM_2 \ell^{M_1 L}}{M_2 \ell^{M_2 L} + M_1 \ell^{M_1 L}} \ell^{\frac{-\lambda^2}{v}t}\left[\ell^{M_1 x} + \frac{M_1 \ell^{M_1 L}}{M_2 \ell^{M_2 L}} \ell^{M_2 x}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (11)$$

The developed expression in [11] considered two conditions through the rate of migration under change of state, concentration increasing or decreasing with respect to distances on the velocity rate of flow. The change of state and time were combined to develop the expression stated in [11]. The rate of dispersions were monitored base on the rate of wind velocities considered in the system, the application of quadratic function at these level of derived solution were imperative to integrate these parameters in terms of their various function base on their relationship.

$$V\frac{\partial c}{\partial y} = Dy\frac{\partial^2 c}{\partial y^2} - U\frac{\partial c}{\partial y} + K_d = -\beta^2$$

$$\Rightarrow VT^{1} - \lambda^{2} \ \ and \ \ Dy\,Y^{11} - UY^{1} + \left(K_{d} + \beta^{2}\right) = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (12)$$

$$T = A_{1}\,\ell^{\frac{-\lambda^{2}}{v}t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (13)$$

$$Y = B_{1}\,Cos\,n_{1}y + B_{2}\,Sin\,n_{2}y \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (14)$$

So that, we combine equations (13) and (14) yield

$$C(y,t) = A_{1}\,\ell^{\frac{-\lambda^{2}}{v}t}\left(B_{1}\,Cos\,n_{1}y + B_{2}\,Sin\,n_{2}y\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (15)$$

Subject equation (15) to equation (5), yield: -

$$C(y,t) = \frac{CoM_{2}\ell^{M_{2}L}}{M_{2}\ell^{M_{2}L} + M_{1}\ell^{M_{1}L}}\,\ell^{\frac{-\lambda^{2}}{v}t}\left[b\,Cos\frac{n\pi}{H}y\right], n = 1, 2, 3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (16)$$

Summary over a Fourier series, in the section [O, H], we have

$$C(y,t) = \frac{Co\,M_{2}\ell^{M_{2}L}}{M_{2}\ell^{M_{2}L} + M_{1}\ell^{M_{1}L}}\,\ell^{\frac{-\lambda^{2}}{v}t}\left[\frac{a_{o}}{2} + \sum_{n=1}^{\infty} a_{1}\,Cos\frac{n\pi}{H}y\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (17)$$

$$\Rightarrow \frac{a_{o}}{2} + \frac{1}{H}\int_{o}^{H} f(u)\,du, \ \ b_{1}\,\ell\,\frac{2}{H}\int_{o}^{H} f(u)\,Cos\,n\,udu$$

Hence, $C(y,t) = \frac{1}{H}\int_{o}^{H} f(u)\,du + \frac{2}{H}\sum Cos\frac{n\pi}{H}y\int_{o}^{H} f(u)\,Cos\,n\,udu$ ……… (18)

Hence bo = a_o and b_n = a_n, for $n \geq 1$.

At this point, the pollutant is following with a uniform underground velocity with transverse dispersion.

Substituting equation (11) and (18) into equation (6), so that we have: -

$$C(x,y,t) = \frac{Co\,M_{2}\ell^{M_{2}L}}{M_{2}\ell^{M_{2}L} + M_{1}\ell^{M_{1}L}}\,\ell^{\frac{-\lambda^{2}}{v}t}\left[\ell^{M_{1}x} + \frac{M_{1}\ell^{M_{1}L}}{M_{2}\ell^{M_{2}L}}\,\ell^{M_{2}x} + bn\,Cos\frac{n\pi}{H}y\right] \qquad \ldots\ldots\ldots \qquad (19)$$

As $x = \to\infty,\ y \to 0,\ c \to 0$ so that we have

$$C(x,y,t) = \frac{Co\,M_{2}\ell^{M_{2}L}}{M_{2}\ell^{M_{2}L} + M_{1}\ell^{M_{1}L}}\,\ell^{\frac{-\lambda^{2}}{v}t}\left[\ell^{M_{1}x} + bo + \sum_{n=1}^{\infty} bn\,Cos\frac{n\pi}{H}y\right] \qquad \ldots\ldots\ldots \qquad (20)$$

The expression in twenty is the derived model for the deposition of potassium in Ntawogba creek, the study were able to determined various parameters that should influences the system as observed in the study area, point sources discharge were predominant discharge of biological waste at various stations discharging direct to the creek, the concept of these study was to monitor the sources and rate of concentration in Ntawogba creek, this also include velocity of flow at different conditions considered in the system. The derived solution express their various behaviour in the transport system, the rate of flow express the level of dispersions at vertical direction, flow rate of pollution at any point of discharge through the drains were considered thus express their rate of pressure in the creek. The express model will definitely predict the rate concentration of this microelement in Ntawogba creek.

4. Conclusion

The discharge of biological waste call for intensive studies on regular bases due to ignorance of environmental base line, these are of serious concern on human health, several environmental laws has been developed by our experts in government but nobody cares about it implementation for thorough health safety in our environments. The behaviour of human settlements on health safety in our environment is another deficiency that studies should be thorough carried for serious orientation. The pollution discharges without regulations are base on these deficiencies in environmental sanitation. Monitoring the rate of potassium in the creek was to determine the level of its concentration; these microelements cause increase of microbial population in surface water. The rate of migration will be observed through the developed model for the study.

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